

Nucleon properties in nuclear matter

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We present recent studies on the effective mass of the nucleon in infinite and homogeneous nuclear matter and its relation to nuclear matter properties within the framework of the in-medium modified Skyrme model. Medium modifications are achieved by introducing optical potential for pion fields and parametrization of the Skyrme parameter in nuclear medium. The present approach is phenomenologically well justified by pion physics in nuclear matter and describe successfully bulk nuclear matter properties.

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I. INTRODUCTION

The equation of state (EOS) has been one of the most important topics in nuclear many-body problems. In-medium modified Skyrme model [1] and especially its recently modified version [2] allow us to investigate not only single nucleon properties in nuclear matter, but also the matter as a whole [1, 2][9]. In its initial version, this approach has been used to investigate the properties of the single nucleon in nuclear matter, pionic clouds [1] being considered in determining external parameters. As a more realistic approach, we need to modify the core part of the nucleon, which provides a possibility to reproduce the binding energy of the system and to discuss the thermodynamic properties of the bulk matter [2]. While the core of the nucleon in the Skyrme model is represented by the Skyrme term in the Lagrangian, its modification is related to the change of the Skyrme parameter. In more detailed treatments, Skyrme's quartic stabilizing term may be revised by explicit vector meson degrees of freedom. In this sense, the core modifications of the nucleon may be pertinent to the vector meson properties in nuclear matter. In the present contribution we will describe the peculiarities of our approach.

In Ref. [1], the initial in-medium modified Skyrme Lagrangian was presented, in which the modifications were achieved by changing the pion mass in nuclear medium by means of the nonlocal optical potential. Due to the nonlocality, not only the mass term but also the kinetic term in the Lagrangian were changed. That procedure does not affect the soliton core, while modifications were exact only in linear approximation and by assumption it was extrapolated to the nonlinear case. In general, one should take into account also the modifications of the quartic term. This is especially required in the case of high density considerations. In this regard, we need to modify also the Skyrme's stabilizing term, i.e., to change the Skyrme parameter as a function of the external nuclear density. The resulting Lagrangian is given as follows:

$$\begin{aligned} \mathcal{L}^* = & \frac{F_\pi^2}{16} \text{Tr} \left(\frac{\partial U}{\partial t} \right) \left(\frac{\partial U^\dagger}{\partial t} \right) - \frac{F_\pi^2}{16} \alpha_p(\vec{r}) \text{Tr}(\vec{\nabla} U) \cdot (\vec{\nabla} U^\dagger) \\ & + \frac{1}{32e^2\gamma(\vec{r})} \text{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 + \frac{F_\pi^2 m_\pi^2}{16} \alpha_s(\vec{r}) \text{Tr}(U + U^\dagger - 2), \end{aligned} \quad (1)$$

where F_π denotes the pion decay constant, e is the Skyrme parameter, and m_π stands for the pion mass. The medium functionals, α_s , α_p and γ , are written in the following forms

$$\alpha_s = 1 - \frac{4\pi b_0 \rho(\vec{r}) f}{m_\pi^2}, \quad \alpha_p = 1 - \frac{4\pi c_0 \rho(\vec{r})}{f + g'_0 4\pi c_0 \rho(\vec{r})}, \quad \gamma = \exp \left(-\frac{\gamma_{\text{num}} \rho(\vec{r})}{1 + \gamma_{\text{den}} \rho(\vec{r})} \right). \quad (2)$$

They represent the influence of the surrounding environment on the properties of the single skyrmion. The parameters α_s and α_p are related to the corresponding phenomenological S - and P -wave pion-nucleus scattering lengths and volumes, i.e. b_0 and c_0 , respectively, and describe the pion physics in a baryon-rich environment [3]. The last functional γ represents the modifications of the skyrme parameter with two variational parameters γ_{num} and γ_{den} [2].

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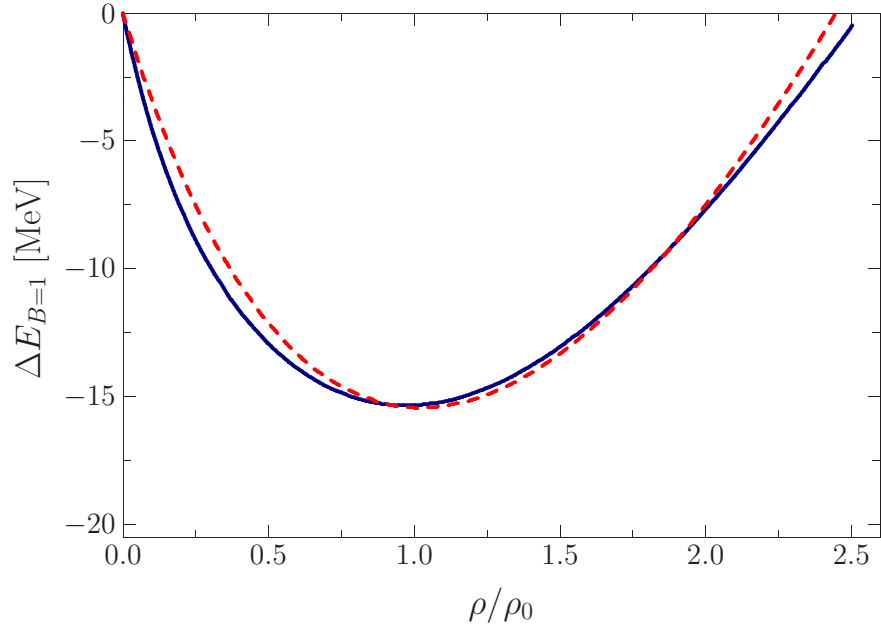


FIG. 1: The binding energy per nucleon as a function of ρ/ρ_0 . The solid curve corresponds to the results with $\gamma_{\text{num}} = 2.1m_\pi^{-3}$, $\gamma_{\text{den}} = 1.45m_\pi^{-3}$ and P -wave scattering volume $c_0 = 0.21m_\pi^{-3}$. The dashed one draws the case when $\gamma_{\text{num}} = 0.8m_\pi^{-3}$, $\gamma_{\text{den}} = 0.5m_\pi^{-3}$ and P -wave scattering volume $c_0 = 0.09m_\pi^{-3}$. The S -wave scattering length has the value $b_0 = -0.024m_\pi^{-1}$. The correlation parameter is fitted near its experimental value $g'_0 = 0.7$.

The density of the surrounding nuclear environment is given by ρ , g'_0 denotes the Lorentz-Lorenz or correlation parameter, $f = 1 + m_\pi/m_N^{\text{free}}$ represents the kinematical factor, and m_N^{free} is the nucleon mass in free space.

The Lagrangian in Eq. (1) will be used to calculate properties of the nucleons in nuclear matter and the bulk matter properties. The parameters of the model are fitted to be $F_\pi = 108.78$ MeV and $e = 4.85$ so as to reproduce the experimental values of the nucleon and Δ in free space. The pion mass is also fixed to be its experimental value, $m_\pi = m_\pi^{\text{exp}} = 134.98$ MeV. A set of values of parameters in the medium functionals (2) are taken from the analysis of phenomenological data for pion-nucleus scattering [3]. Since the environment acting on the single nucleon properties has a homogeneous and constant density, one can choose the spherically symmetric “hedgehog” form for the boson field $U = \exp\{i\hat{n} \cdot \vec{\tau}F(r)\}$, where \hat{n} denotes the unit vector in coordinate space and $\vec{\tau}$ are the usual Pauli matrices. Then the problem will be much simplified and thereafter we will follow this choice.

II. NUCLEAR MATTER PROPERTIES

The results of the binding energy per nucleon are shown in Fig. 1. The solid and dashed curves draw the parametrization of γ in Eq. (2). Our minimization procedure has been performed in such a way that the values of the variational parameters, i.e., γ_{num} and γ_{den} , lead to the minimum of the binding energy per nucleon, defined crudely as

$$\Delta E_{B=1} = m_N^*(\rho) - m_N^{\text{free}}, \quad (3)$$

at the normal nuclear matter density. The difference between the two curves in Fig 1 arise from the different values of the P -wave scattering length, i.e. $c_0 = 0.21m_\pi^{-3}$ corresponds to the solid curve and $c_0 = 0.09m_\pi^{-3}$ to the dashed one. One can see that the dependence on the density is rather insensitive to the changes of P -wave scattering volume. The effect of the changes in b_0 is even smaller.

However, another important quantity is the compressibility of nuclear matter defined as

$$K = 9\rho_0^2 \left. \frac{\partial^2 \Delta E_{B=1}}{\partial \rho^2} \right|_{\rho=\rho_0}, \quad (4)$$

which depends strongly on c_0 . The corresponding result is presented in Fig. 2. For example, at the empirical value of $c_0 = 0.21m_\pi^{-3}$, the compressibility turns out to be very large ($K \sim 1640$ MeV). The other approaches like relativistic Dirac-Brueckner-Hartree-Fock [4, 5] or the Walecka model [6] give much more smaller values of the compressibility.

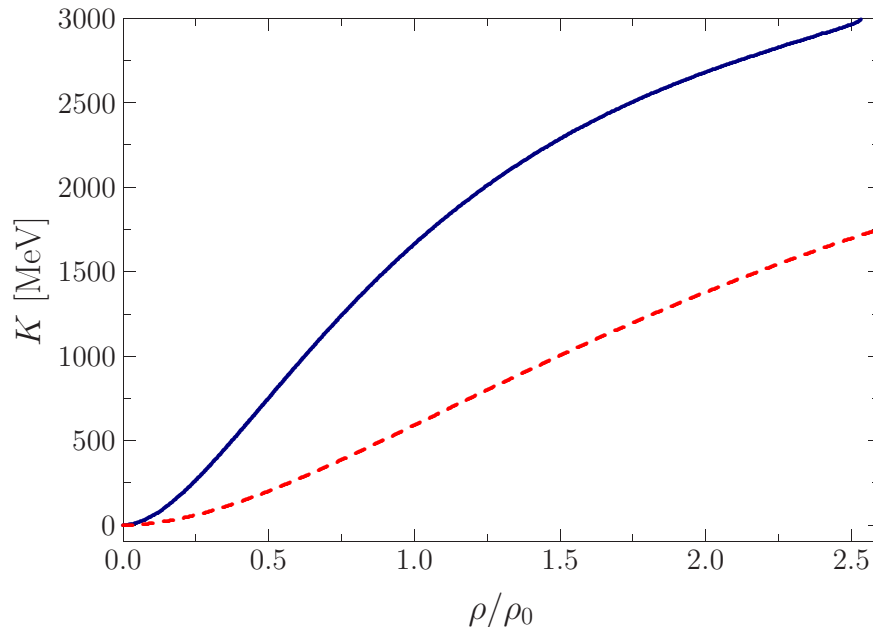


FIG. 2: The compressibility as a function of ρ/ρ_0 . The other notations are the same as in Fig. 1.

Lowering the value of c_0 leads to the noticeably decreasing value of K : if we use $c_0 = 0.09m_\pi^{-3}$, the compressibility decreases and becomes consistent with that of the Walecka model ($K \sim 580$ MeV). Lowering further the value up to $c_0 = 0.06m_\pi^{-3}$ give the result $K \sim 300$ MeV that is comparable with experimental value of K and with that in Dirac-Brueckner-Hartree-Fock approaches. Our conclusion is that the smaller values of c_0 than that used in the pionic atom analysis will be preferable as far as the compressibility is concerned. The compressibility K is sensitive to the position of the saturation point. If one fits the saturation point at slightly lower densities, it is found that the compressibility drastically decreases.

One can also discuss the symmetry energy in a crude approximation [7, 8]

$$E_{\text{sym}} = \frac{1}{12} m_{\Delta-N}^*, \quad (5)$$

where $m_{\Delta-N}^*$ is the effective $\Delta - N$ mass difference. We obtained the following results for the symmetry energy: $E_{\text{sym}}(\rho_0) \approx 14.19$ MeV for $c_0 = 0.09m_\pi^{-3}$ whereas $E_{\text{sym}}(\rho_0) \approx 8.71$ MeV for $c_0 = 0.21m_\pi^{-3}$. These results slightly are underestimated in comparison with its experimental value $E_{\text{sym}} \sim 20 - 30$ MeV. Further generalizations of the model including the isospin breaking effects, finite nuclei corrections may improve the situation.

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